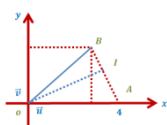


السؤال الثالث:

$$Z_A = 4 \Rightarrow A(4,0) \quad -1$$

$$Z_B = 2\sqrt{2} + 2\sqrt{2}i \Rightarrow B(2\sqrt{2}, 2\sqrt{2})$$



$$Z_B = 2\sqrt{2} + 2\sqrt{2}i$$

$$\Rightarrow r = \sqrt{8+8} = 4$$

$$\cos \theta = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} \quad \sin \theta = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} \quad \theta = \frac{\pi}{4}$$

$$\Rightarrow Z_B = 4e^{i\frac{\pi}{4}}$$

$$-2 \quad |Z_A| = |Z_B| = 4 \quad \text{بما أن:}$$

المثلث متساوي الساقين.

بما أن $[OI]$ متوسط في مثلث متساوي الساقين رأسه

$O \Leftarrow [OI]$ منتصف داخلي للزاوية $\hat{O}$ في المثلث

(OAB)

$$\Rightarrow (\vec{u}, \vec{OI}) = \frac{1}{2} \left(\frac{\pi}{4} \right) = \frac{\pi}{8}$$

$$-3 \quad Z_I = \frac{Z_A + Z_B}{2} = \frac{4 + 2\sqrt{2} + 2\sqrt{2}i}{2}$$

$$Z_I = (2 + \sqrt{2}) + \sqrt{2}i$$

لنوجد Z_I بالشكل الأسّي:

$$r = \sqrt{(2 + \sqrt{2})^2 + (\sqrt{2})^2}$$

$$= \sqrt{4 + 4\sqrt{2} + 2 + 2}$$

$$= \sqrt{8 + 4\sqrt{2}}$$

$$r = \sqrt{4(2 + \sqrt{2})} = 2\sqrt{2 + \sqrt{2}}$$

$$\theta = \frac{\pi}{8} \quad \text{حسب الطلب السابق:}$$

$$Z_I = 2\sqrt{2 + \sqrt{2}} e^{i\frac{\pi}{8}}$$

$$- \quad \text{استنتاج } \sin \frac{\pi}{8}$$

$$Z_I = 2\sqrt{2 + \sqrt{2}} \left[\cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right]$$

$$= 2\sqrt{2 + \sqrt{2}} \cos \frac{\pi}{8} + i 2\sqrt{2 + \sqrt{2}} \sin \frac{\pi}{8}$$

بالمقارنة بين الشكلين الجبري و المثلثي لـ Z_I نجد:

$$2\sqrt{2 + \sqrt{2}} \sin \frac{\pi}{8} = \sqrt{2}$$

$$\Rightarrow \sin \frac{\pi}{8} = \frac{\sqrt{2}}{2\sqrt{2 + \sqrt{2}}}$$

السؤال الأول:

$$P(Z) = Z^2 + (1 + 2i)Z + 3 + 3i \quad -1$$

$$P(Z_A) = (-1 + i)^2 + (1 + 2i)(-1 + i) + 3 + 3i$$

$$= 1 - 2i - 1 - 1 + i - 2i - 2 + 3 + 3i$$

$$= 0 \quad \text{"محققة"}$$

$$Z_1 + Z_2 = -\frac{b}{a} \quad \text{الاستنتاج:}$$

$$\Rightarrow -1 + i + Z_2 = -1 - 2i$$

$$\Rightarrow Z_2 = -3i = Z_B$$

$$-2 \quad (Z' - Z_B) = e^{i\theta} (Z_A - Z_B)$$

$$Z' = e^{i\frac{\pi}{2}} (Z_A - Z_B) + Z_B$$

$$= i(-1 + i + 3i) - 3i$$

$$= i(-1 + 4i) - 3i$$

$$Z' = -i - 4 - 3i = -4 - 4i$$

$$-3 \quad Z_A = -1 + i$$

$$r = \sqrt{1+1} = \sqrt{2}$$

$$\cos \theta = -\frac{1}{\sqrt{2}} \quad \sin \theta = 1/\sqrt{2} \quad \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$Z_A = \sqrt{2} e^{i(\frac{3\pi}{4})}$$

السؤال الثاني:

$$-1 \quad \frac{b-a}{c-a} = \frac{-6+3i-6+i}{-18+7i-6+i}$$

$$= \frac{-12+4i}{-24+8i} = \frac{-12+4i}{2(-12+4i)}$$

$$\frac{b-a}{c-a} = \frac{1}{2}$$

$$\arg \left(\frac{b-a}{c-a} \right) = \arg \left(\frac{1}{2} \right)$$

$$(\vec{AC}, \vec{AB}) = 2\pi$$

$C, B, A \Leftarrow$ تقع على استقامة واحدة.

$$-2 \quad Z_D = e^{i\theta} Z_A$$

$$\Rightarrow e^{i\theta} = \frac{Z_D}{Z_A} = \frac{1+6i}{6-i}$$

$$= \frac{1+6i}{6-i} \cdot \frac{6+i}{6+i} = \frac{6+i+36i-6}{36+1}$$

$$= \frac{37i}{37} = i = e^{i\frac{\pi}{2}} \Rightarrow \theta = \frac{\pi}{2}$$

$\Leftarrow$ المثلث DOA قائم في O ومتساوي الساقين.

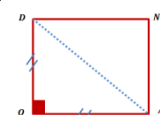
$$-3 \quad \text{الشرط: } Z_{\overline{DN}} = Z_{\overline{OA}}$$

$$Z_N - Z_D = Z_A - Z_O$$

$$Z_N = Z_A + Z_D$$

$$= 6 - i + 1 + 6i$$

$$Z_N = 7 + 5i$$



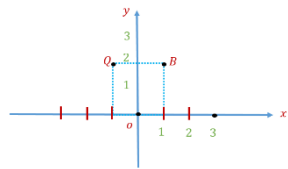
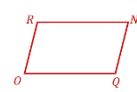
السؤال الخامس:

السؤال الرابع:

$Z_A = 1 + 3i, Z_B = 1 + 2i \quad -1$ $Z_A \left[\begin{array}{l} r = \sqrt{1+9} = \sqrt{10} \\ \theta = \alpha \end{array} \right] \Rightarrow Z_A = \sqrt{10}e^{i\alpha}$ $Z_B \left[\begin{array}{l} r = \sqrt{1+4} = \sqrt{5} \\ \theta = \beta \end{array} \right] \Rightarrow Z_B = \sqrt{5}e^{i\beta}$ $Z_A \cdot Z_B = \sqrt{10}e^{i\alpha} \cdot \sqrt{5}e^{i\beta}$ $= \sqrt{50}e^{i(\alpha+\beta)}$ $= 5\sqrt{2}e^{i(\alpha+\beta)} \quad (1)$
$Z_A \cdot Z_B = (1 + 3i) \cdot (1 + 2i) \quad -2$ $= 1 + 2i + 3i - 6$ $= -5 + 5i = 5(-1 + i)$ لنوجد $Z_A \cdot Z_B$ بالشكل الأسّي: $r = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$ $\left. \begin{array}{l} \cos \theta = \frac{-1}{\sqrt{2}} \\ \sin \theta = \frac{1}{2} \end{array} \right\} \Rightarrow \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $Z_A \cdot Z_B = 5\sqrt{2}e^{i\frac{3\pi}{4}} \quad (2)$
-3- بالمقارنة بين (1) و (2) نجد: $\alpha + \beta = \frac{3\pi}{4}$

انتهى حل الاسئلة

مع أنس أحمد

1. $A(3,0), B(1,2), Q(-1,2)$ 
2. $Z_N = e^{i\theta} Z_A$ $Z_N = e^{i\frac{\pi}{2}}(3) = 3i$
3. الشرط: $Z_{\overline{OR}} = Z_{\overline{QN}}$  $Z_R = Z_N - Z_Q$ $= 3i + 1 - 2i = 1 + i$
4. $\frac{Z_{\overline{AB}}}{Z_{\overline{OR}}} = \frac{Z_B - Z_A}{Z_R - Z_O} = \frac{1+2i-3}{1+i}$ $= \frac{-2+2i}{1+i} \cdot \frac{1-i}{1-i}$ $= \frac{-2+2i+2i+2}{1+1} = \frac{4i}{2} = 2i$ $\arg\left(\frac{Z_B - Z_A}{Z_R - Z_O}\right) = \arg(2i)$ متعامدان $(\overrightarrow{OR}, \overrightarrow{AB}) = \frac{\pi}{2} \Rightarrow$ $\frac{ Z_B - Z_A }{ Z_R - Z_O } = 2i \Rightarrow \frac{AB}{OR} = 2$ $\Rightarrow OR = \frac{1}{2} AB$