

$$\textcircled{5} f(x) = \frac{\sin(2023x)}{\sin(x)} ; a = 0$$

$$\lim_{x \rightarrow 0} f(x) = \frac{0}{0} \quad (\text{حالة عدم تعيين})$$

$$f(x) = \frac{2023x \cdot \frac{\sin(2023x)}{2023x}}{x \cdot \frac{\sin x}{x}}$$

$$\lim_{x \rightarrow 0} f(x) = 2023 \times \frac{1}{1} = 2023$$

$$\textcircled{6} f(x) = \frac{\sin x - \sin 3x}{3x} ; a = 0$$

$$\lim_{x \rightarrow 0} f(x) = \frac{0}{0} \quad (\text{حالة عدم تعيين})$$

$$f(x) = \frac{\sin x}{3x} - \frac{\sin 3x}{3x}$$

$$\lim_{x \rightarrow 0} f(x) = \frac{1}{3}(1) - (1) = -\frac{2}{3}$$

$$\textcircled{7} f(x) = \frac{x \sin x}{1 - \cos^2(2x)} ; a = 0$$

$$f(x) = \frac{x^2 \frac{\sin x}{x}}{4x^2 \frac{\sin^2(2x)}{4x^2}} = \frac{\frac{\sin x}{x}}{4 \left(\frac{\sin 2x}{2x} \right)^2}$$

$$\lim_{x \rightarrow 0} f(x) = \frac{1}{4(1)^2} = \frac{1}{4}$$

$$\textcircled{8} f(x) = \frac{x \sin(3x)}{x^2 - 2 \sin^2(x)} ; a = 0$$

$$f(x) = \frac{3x^2 \frac{\sin 3x}{3x}}{x^2 \left(1 - 2 \frac{\sin^2 x}{x^2} \right)} = \frac{3 \frac{\sin 3x}{3x}}{1 - 2 \left(\frac{\sin x}{x} \right)^2}$$

$$\lim_{x \rightarrow 0} f(x) = \frac{3(1)}{1 - 2(1)^2} = \frac{3}{-1} = -3$$

$$\textcircled{9} f(x) = \frac{1 - \cos^2 x}{4x \sin(x)} ; a = 0$$

$$f(x) = \frac{\sin^2 x}{4x \sin x} = \frac{x^2 \frac{\sin^2 x}{x^2}}{4x^2 \frac{\sin x}{x}} = \frac{\left(\frac{\sin x}{x} \right)^2}{4 \frac{\sin x}{x}}$$

$$\lim_{x \rightarrow 0} f(x) = \frac{(1)^2}{4(1)} = \frac{1}{4}$$

$$\textcircled{1} f(x) = \frac{2 \sin x}{3x} ; a = 0$$

$$\lim_{x \rightarrow 0} f(x) = \frac{0}{0} \quad (\text{حالة عدم تعيين})$$

$$\lim_{x \rightarrow 0} f(x) = \frac{2}{3} \cdot \frac{\sin x}{x} = \frac{2}{3} \times 1 = \frac{2}{3}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\textcircled{2} f(x) = \frac{3 \sin(2x)}{-5x} ; a = 0$$

$$\lim_{x \rightarrow 0} f(x) = \frac{0}{0} \quad (\text{حالة عدم تعيين})$$

$$f(x) = \frac{3}{-5} \cdot \frac{2 \sin 2x}{2x}$$

$$\lim_{x \rightarrow 0} f(x) = -\frac{3}{5} \times 2 \times 1 = -\frac{6}{5}$$

$$\textcircled{3} f(x) = \frac{\sin 3x}{\sin 4x} ; a = 0$$

$$\lim_{t \rightarrow 0} f(x) = \frac{0}{0} \quad (\text{حالة عدم تعيين})$$

$$f(x) = \frac{3x \frac{\sin 3x}{3x}}{4x \frac{\sin 4x}{4x}}$$

$$= \frac{3}{4} \cdot \frac{\frac{\sin 3x}{3x}}{\frac{\sin 4x}{4x}}$$

$$\lim_{x \rightarrow 0} f(x) = \frac{3}{4} \cdot \frac{1}{1} = \frac{3}{4}$$

$$\textcircled{4} f(x) = \frac{x + \tan x}{x + \sin x} ; a = 0$$

$$\lim_{x \rightarrow 0} f(x) = \frac{0}{0} \quad (\text{حالة عدم تعيين})$$

$$\lim_{x \rightarrow 0} \frac{x + \tan x}{x + \sin x} = \lim_{x \rightarrow 0} \frac{x \left(1 + \frac{\tan x}{x} \right)}{x \left(1 + \frac{\sin x}{x} \right)}$$

$$= \frac{1+1}{1+1} = \frac{2}{2} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan(x)}{x} = 1$$

$$= \frac{2\sin^2(2x)}{4x^2 \left(\frac{3x^3}{4x^2} - \frac{4x}{4x^2} \right)} = \frac{2\sin^2(2x)}{\left(\frac{3x}{2} - \frac{1}{x} \right) 4x^2}$$

$$= \frac{2}{\frac{3x}{2} - \frac{1}{x}} \cdot \left(\frac{\sin 2x}{2x} \right)^2$$

$$\lim_{x \rightarrow 0^+} f(x) = \frac{2}{-\infty} (1)^2 = 0$$

$$\lim_{x \rightarrow 0^-} f(x) = \frac{2}{+\infty} (1)^2 = 0$$

$$\textcircled{15} f(x) = \frac{\cos 3x - \cos 4x}{x \sin(2x)} ; a = 0$$

$$= \frac{-2\sin\left(\frac{3x+4x}{2}\right)\sin\left(\frac{3x-4x}{2}\right)}{x \sin 2x}$$

$$= \frac{\cancel{2} \frac{7x}{2} \frac{\sin\left(\frac{7x}{2}\right)}{\frac{7x}{2}} \cdot \cancel{\frac{x}{2}} \left(\frac{\sin\left(-\frac{x}{2}\right)}{-\frac{x}{2}} \right)}{\cancel{2x} \cancel{\frac{\sin 2x}{2x}}}$$

$$\lim_{x \rightarrow 0} f(x) = \frac{7}{4} \frac{(1)(1)}{(1)} = \frac{7}{4}$$

$$\textcircled{16} f(x) = \frac{\cos^2 x - \cos x}{1 - \cos x} ; a = 0$$

$$= -\frac{\cos x(1 - \cos x)}{1 - \cos x}$$

$$\lim_{x \rightarrow 0} f(x) = -1$$

$$\textcircled{17} f(x) = \frac{2\cos x - 2}{\sqrt{4x^2 + 2} - \sqrt{2}} ; a = 0$$

$$= \frac{-2(1 - \cos x)}{\sqrt{4x^2 + 2} - \sqrt{2}}$$

$$= \frac{-2\left(2\sin^2\frac{x}{2}\right)(\sqrt{4x^2 + 2} + \sqrt{2})}{(\sqrt{4x^2 + 2} - \sqrt{2})(\sqrt{4x^2 + 2} + \sqrt{2})}$$

$$= \frac{-4\sin^2\frac{x}{2}(\sqrt{4x^2 + 2} + \sqrt{2})}{\cancel{4} \frac{4x^2 + 2}{4} \cancel{-2}}$$

$$= \frac{-1}{4} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 (\sqrt{4x^2 + 2} + \sqrt{2})$$

$$\lim_{x \rightarrow 0} f(x) = -\frac{1}{4} (1)^2 (2\sqrt{2}) = -\frac{\sqrt{2}}{2}$$

$$\textcircled{10} f(x) = \frac{\tan x - \sin x}{\sin^3 x} ; a = 0$$

$$= \frac{\frac{\sin x}{\cos x} - \sin x}{\sin^3 x}$$

$$= \frac{\cancel{\sin x} \left(\frac{1}{\cos x} - 1 \right)}{\frac{\sin^3 x}{\sin^2 x}} = \frac{\frac{1 - \cos x}{\cos x}}{\frac{\sin^2 x}{\sin^2 x \cos x}} = \frac{1 - \cos x}{\sin^2 x \cos x}$$

$$= \frac{2\sin^2\left(\frac{x}{2}\right)}{\sin^2 x \cos x} = \frac{\cancel{2} \frac{x^2}{2} \frac{\sin^2 \frac{x}{2}}{\frac{x^2}{4}}}{x^2 \frac{\sin^2 x}{x^2} \cos x}$$

$$= \frac{1}{2} \cdot \frac{\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2}{\left(\frac{\sin x}{x} \right)^2 \cos x}$$

$$\lim_{x \rightarrow 0} f(x) = \frac{1}{2} \frac{(1)^2}{(1)^2 \cdot 1} = \frac{1}{2}$$

$$\textcircled{11} f(x) = \frac{\cos(12x) - 1}{72x^2} ; a = 0$$

$$= \frac{-(1 - \cos(2x))}{72x^2} = \frac{-\cancel{2} \sin^2(6x)}{\cancel{72} x^2}$$

$$= -\left(\frac{\sin 6x}{6x} \right)^2$$

$$\lim_{x \rightarrow 0} f(x) = -(1)^2 = -1$$

$$\textcircled{12} f(x) = \frac{6x^2 + 2 - 2\cos(2x)}{2x^2} ; a = 0$$

$$= \frac{3\cancel{6}x^2}{\cancel{2}x^2} + \frac{2(1 - \cos 2x)}{2x^2}$$

$$= 3 + \frac{2\sin^2 x}{x^2}$$

$$= 3 + 2 \left(\frac{\sin x}{x} \right)^2$$

$$\lim_{x \rightarrow 0} f(x) = 3 + 2(1)^2 = 5$$

$$\textcircled{13} f(x) = \frac{\sin(x-1)}{x^2 + x - 2} ; a = 1$$

$$= \frac{\sin(x-1)}{(x+2)(x-1)} = \frac{1}{x+2} \cdot \frac{\sin(x-1)}{x-1}$$

$$\lim_{x \rightarrow 1} f(x) = \frac{1}{1+2} \times (1) = \frac{1}{3}$$

$$\textcircled{14} f(x) = \frac{1 - \cos(4x)}{6x^3 - 4x} ; a = 0$$