التمرين الأول:

أوجد نهاية التابع f عند a :

$f(x)=\dfrac{x^2-3x-2}{x-1}\ ,a=1$	$f(x)=\dfrac{x-1}{x^2-4x+3}\ ,a=1$
$f(x)=\dfrac{\sqrt{x}-1}{x-1}\ ,a=1$	$f(x)=\dfrac{\sqrt{x+1}-1}{x^2-x}\ ,a=0$
$f(x)=\dfrac{\sqrt{4-x}-\sqrt{4+x}}{x}\ ,a=0$	$f(x)=\dfrac{2-\sqrt{x+2}}{3-\sqrt{2x+5}}\ ,a=2$

$f(x)=\dfrac{x^2-9}{\sqrt{x}-\sqrt{3}}\ ,a=3\ ,+\infty$	$f(x)=\sqrt{5x+1}-x\ ,a=+\infty$
$f(x)=\sqrt{x^2+x+1}-x\ ,a=-\infty$	$f(x)=\dfrac{\sqrt{x^2+1}-\sqrt{2}}{x-1}\ a=1$
$f(x)=x\left(\sqrt{4+\dfrac{1}{x}}-2\right)\ ,a=0^+$	$f(x)=\dfrac{\sqrt{x^2+1}-x}{\sqrt{x^2-1}-\sqrt{x^2-2}}\ ,a=+\infty$

$f(x)=\dfrac{3x^2-3}{x-1}\ ,a=1$	$f(x)=\sqrt{2x^2-1}-3x$ $a=+\infty$
$f(x)=\sqrt{3x^2+2x+2}-\sqrt{3x+1}$ $a=+\infty$	$f(x)=\sqrt{x^2-1}-x$ $a=+\infty$
$f(x)=\dfrac{\sqrt{x^2+3}}{2x}$ $a=-\infty$	$f(x)=\sqrt{9x^2+2x-1}-2x+1$ $a=+\infty$
$f(x)=\dfrac{x\sqrt{x}-2\sqrt{2}}{\sqrt{x}-\sqrt{2}}$ $a=2\ ,+\infty$	$f(x)=\dfrac{x}{\sqrt{x+1}}-\dfrac{x}{\sqrt{x+2}}$ $a=+\infty$
$f(x)=x^2\left(\sqrt{2+\dfrac{1}{x}}-\sqrt{2}\right)$ $a=+\infty$	$f(x)=\dfrac{\sqrt{x+1}-3}{16-2x}$ $a=8$

$f(x)=\dfrac{x^2-x}{sinx}\ ,a=0$	$f(x)=sin\,x\sqrt{1+\dfrac{1}{x^2}}\ ,a=0^+$
$f(x)=\dfrac{sin\left(\pi\sqrt{1-x}\right)}{x}\ ,a=0$	$f(x)=\dfrac{sin(2x)}{3x}\ ,a=0$
$f(x)=\dfrac{sinx}{\sqrt{x^2+x}}\ ,a=0$	$f(x)=\dfrac{sin(\pi x)}{sin(2x)}\ ,a=0$
$f(x)=\dfrac{\sqrt{x^2+4}-2}{x.\,sin3x}\ ,a=0$	$f(x)=\dfrac{2x^2+sinx}{x}\ ,a=0$

التمرين الثاني:

ليكن لدينا التابع f المعرف على R بالعلاقة:

$$f(x)=\frac{x}{|x|+3}$$

١. احسب نهاية التابع f عند $-\infty$

٢. أوجد عدداً A يحقق الشرط إذا كان $x<A$ كان $f(x)$ من المجال $]-1.05,-0.95[$

التمرين الثالث:

ليكن لدينا التابع f المعين بالعلاقة:

$$f(x)=\frac{x+3}{x-3}$$

١. أوجد نهاية التابع f عند الـ 5

٢. عين مجالاً I مركزه 5 يحقق الشرط: إذا كان x من المجال I كان $f(x)$ من المجال $J=]3.95,4.05[$

التمرين الرابع:

ليكن لدينا التابع f المعرف على $]-5,+\infty[$ وفق:

$$f(x)=\frac{2x+1}{x+5}$$

١. احسب $\lim\limits_{x\to+\infty}f(x)$ واستنتج $\lim\limits_{x\to+\infty}f\big(f(x)\big)$

٢. جد عدداً حقيقياً يحقق الشرط: إذا كان $x>A$ كان $f(x)$ من المجال $]1.99,2.01[$

التمرين الأول:

أوجد نهاية التابع f عند a :

$f(x)=\dfrac{x^2-3x-2}{x-1}\ ,a=1$	$f(x)=\dfrac{x-1}{x^2-4x+3}\ ,a=1$
$f(x)=\dfrac{\sqrt{x}-1}{x-1}\ ,a=1$	$f(x)=\dfrac{\sqrt{x+1}-1}{x^2-x}\ ,a=0$
$f(x)=\dfrac{\sqrt{4-x}-\sqrt{4+x}}{x}\ ,a=0$	$f(x)=\dfrac{2-\sqrt{x+2}}{3-\sqrt{2x+5}}\ ,a=2$

$$f(x)=\frac{x^2-3x-2}{x-1}$$

$$\lim_{x\to1^-}f(x)=\frac{-4}{0^-}=+\infty$$

$$\lim_{x\to1^+}f(x)=\frac{-4}{0^+}=-\infty$$

$$f(x)=\frac{x-1}{x^2-4x+3}$$

$$\lim_{x\to1}f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{x-1}{x^2-4x+3}$$

$$f(x)=\frac{x-1}{(x-3)(x-1)}$$

$$=\frac{1}{x-3}$$

$$\lim_{x\to1}\mathrm{f(x)}=-\frac{1}{2}$$

$$f(x)=\frac{\sqrt{x}-1}{x-1}$$

$$\lim_{x\to1}f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{\sqrt{x}-1}{x-1}$$

$$=\frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$=\frac{1}{\sqrt{x}+1}$$

$$\lim_{x\to1}f(x)=\frac{1}{2}$$

$$f(x)=\frac{\sqrt{x+1}-1}{x^2-x}$$

$$\lim_{x\to 0} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{\sqrt{x+1}-1}{x^2-x}$$

$$=\frac{\left(\sqrt{x+1}-1\right)\left(\sqrt{x+1}+1\right)}{x^2-x\left(\sqrt{x+1}+1\right)}$$

$$=\frac{x+1-1}{x^2-x\left(\sqrt{x+1}+1\right)}$$

$$=\frac{x}{x(x-1)\left(\sqrt{x+1}+1\right)}$$

$$=\frac{1}{(x-1)\left(\sqrt{x+1}+1\right)}$$

$$\lim_{x\to 0} f(x)=-\frac{1}{2}$$

$$f(x)=\frac{\sqrt{4-x}-\sqrt{4+x}}{x}$$

$$\lim_{x\to 0} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{\sqrt{4-x}-\sqrt{4+x}}{x}$$

$$=\frac{\left(\sqrt{4-x}-\sqrt{4+x}\right)\left(\sqrt{4-x}+\sqrt{4+x}\right)}{x\left(\sqrt{4-x}+\sqrt{4+x}\right)}$$

$$=\frac{4-x-4-x}{x\left(\sqrt{4-x}+\sqrt{4+x}\right)}$$

$$=\frac{-2x}{x\left(\sqrt{4-x}+\sqrt{4+x}\right)}$$

$$=\frac{-2}{\sqrt{4-x}+\sqrt{4+x}}$$

$$\lim_{x\to 0} f(x)=\frac{-2}{4}=-\frac{1}{2}$$

$$f(x)=\frac{2-\sqrt{x+2}}{3-\sqrt{2x+5}}$$

$$\lim_{x\to 2} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{2-\sqrt{x+2}}{3-\sqrt{2x+5}}$$

$$=\frac{\left(2-\sqrt{x+2}\right)\left(2+\sqrt{x+2}\right)\left(3+\sqrt{2x+5}\right)}{\left(3-\sqrt{2x+5}\right)\left(3+\sqrt{2x+5}\right)\left(2+\sqrt{x+2}\right)}$$

$$=\frac{4-x-2}{9-2x-5}.\left(\frac{3+\sqrt{2x+5}}{2+\sqrt{x+2}}\right)$$

$$=\frac{2-x}{4-2x}.\left(\frac{3+\sqrt{2x+5}}{2+\sqrt{x+2}}\right)$$

$$=\frac{2-x}{2(2-x)}.\left(\frac{3+\sqrt{2x+5}}{2+\sqrt{x+2}}\right)$$

$$=\frac{1}{2}.\left(\frac{3+\sqrt{2x+5}}{2+\sqrt{x+2}}\right)$$

$$\lim_{x\to 2} f(x)=\frac{1}{2}.\left(\frac{6}{4}\right)=\frac{6}{8}=\frac{3}{4}$$

$f(x)=\dfrac{x^2-9}{\sqrt{x}-\sqrt{3}}\ ,a=3\,,+\infty$	$f(x)=\sqrt{5x+1}-x\ ,a=+\infty$
$f(x)=\sqrt{x^2+x+1}-x\ ,a=-\infty$	$f(x)=\dfrac{\sqrt{x^2+1}-\sqrt{2}}{x-1}\ \ a=1$
$f(x)=x\left(\sqrt{4+\dfrac{1}{x}}-2\right)\ ,a=0^+$	$f(x)=\dfrac{\sqrt{x^2+1}-x}{\sqrt{x^2-1}-\sqrt{x^2-2}}\ ,a=+\infty$

$$f(x)=\frac{x^2-9}{\sqrt{x}-\sqrt{3}}$$

إيجاد نهاية التابع f عند الـ 3 :

$$\lim_{x\to 3} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{x^2-9}{\sqrt{x}-\sqrt{3}}$$

$$=\frac{(x-3)(x+3)}{\sqrt{x}-\sqrt{3}}$$

$$=\frac{\left(\sqrt{x}-\sqrt{3}\right)\left(\sqrt{x}+\sqrt{3}\right)(x+3)}{\sqrt{x}-\sqrt{3}}$$

$$=\left(\sqrt{x}+\sqrt{3}\right)(x+3)$$

$$\lim_{x\to 3} f(x)=\left(2\sqrt{3}\right)(6)=12\sqrt{3}$$

إيجاد نهاية التابع f عند الـ $+\infty$:

$$\lim_{x\to+\infty} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{\infty}{\infty}$

$$f(x)=\frac{x^2-9}{\sqrt{x}-\sqrt{3}}$$

$$=\frac{x^2\left(1-\frac{9}{x^2}\right)}{x^2\left(\frac{\sqrt{x}}{x^2}-\frac{\sqrt{3}}{x^2}\right)}$$

$$=\frac{1-\frac{9}{x^2}}{\frac{1}{x\sqrt{x}}-\frac{\sqrt{3}}{x^2}}$$

$$\lim_{x\to+\infty} f(x)=\frac{1}{0^+}=+\infty$$

$$f(x)=\sqrt{5x+1}-x$$

$$\lim_{x\to+\infty} f(x)$$

وهي حالة عدم تعيين من الشكل $\infty-\infty$

$$f(x)=\sqrt{5x+1}-x$$

$$=|x|\sqrt{\frac{5}{x}+\frac{1}{x^2}}-x$$

$$=x\sqrt{\frac{5}{x}+\frac{1}{x^2}}-x$$

$$=x\left(\sqrt{\frac{5}{x}+\frac{1}{x^2}}-1\right)$$

$$\lim_{x\to+\infty} f(x)=+\infty(-1)=-\infty$$

$$f(x)=\sqrt{x^2+x+1}+x$$

$$\lim_{x\to-\infty} f(x)$$

وهي حالة عدم تعيين من الشكل $\infty-\infty$

$$f(x)=\sqrt{x^2+x+1}+x$$

$$=\frac{\left(\sqrt{x^2+x+1}+x\right)\left(\sqrt{x^2+x+1}-x\right)}{\sqrt{x^2+x+1}-x}$$

$$=\frac{x^2+x+1-x^2}{\sqrt{x^2+x+1}-x}$$

$$=\frac{x+1}{\sqrt{x^2+x+1}-x}$$

$$=\frac{x+1}{|x|\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-x}$$

$$=\frac{x+1}{-x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-x}$$

$$=\frac{x\left(1+\frac{1}{x}\right)}{x\left(-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-1\right)}$$

$$=\frac{1+\frac{1}{x}}{-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-1}$$

$$\lim_{x\to-\infty} f(x)=-\frac{1}{2}$$

$$f(x)=\frac{\sqrt{x^2+1}-\sqrt{2}}{x-1}$$

$$\lim_{x\to1} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{\sqrt{x^2+1}-\sqrt{2}}{x-1}$$

$$=\frac{\left(\sqrt{x^2+1}-\sqrt{2}\right)\left(\sqrt{x^2+1}+\sqrt{2}\right)}{(x-1)\left(\sqrt{x^2+1}+\sqrt{2}\right)}$$

$$=\frac{x^2+1-2}{(x-1)\left(\sqrt{x^2+1}+\sqrt{2}\right)}$$

$$=\frac{x^2-1}{(x-1)\left(\sqrt{x^2+1}+\sqrt{2}\right)}$$

$$=\frac{(x-1)(x+1)}{(x-1)\left(\sqrt{x^2+1}+\sqrt{2}\right)}$$

$$=\frac{x+1}{\sqrt{x^2+1}+\sqrt{2}}$$

$$\lim_{x\to1} f(x)=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$f(x)=x\left(\sqrt{4+\frac{1}{x}}-2\right)$$

$$\lim_{x\to0^+} f(x)$$

وهي حالة عدم تعيين من الشكل $0(\infty)$

$$f(x)=x\left(\sqrt{4+\frac{1}{x}}-2\right)$$

$$=x.\left(\frac{\left(\sqrt{4+\frac{1}{x}}-2\right)\left(\sqrt{4+\frac{1}{x}}+2\right)}{\sqrt{4+\frac{1}{x}}+2}\right)$$

$$=x.\frac{\left(4+\frac{1}{x}-4\right)}{\sqrt{4+\frac{1}{x}}+2}$$

$$=\frac{x\left(\frac{1}{x}\right)}{\sqrt{4+\frac{1}{x}}+2}$$

$$=\frac{1}{\sqrt{4+\frac{1}{x}}+2}$$

$$\lim_{x\to0^+} f(x)=\frac{1}{\infty}=0$$

$$f(x)=\frac{\sqrt{x^2+1}-x}{\sqrt{x^2-1}-\sqrt{x^2-2}}$$

$$\lim_{x\to+\infty} f(x)$$

وهي حالة عدم تعيين

$$f(x)=\frac{\sqrt{x^2+1}-x}{\sqrt{x^2-1}-\sqrt{x^2-2}}$$

$$f(x)=\frac{\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)\left(\sqrt{x^2-1}+\sqrt{x^2-2}\right)}{\left(\sqrt{x^2-1}-\sqrt{x^2-2}\right)\left(\sqrt{x^2-1}+\sqrt{x^2-2}\right)\left(\sqrt{x^2+1}+x\right)}$$

$$=\frac{x^2+1-x^2}{x^2-1-x^2+2}.\left(\frac{\sqrt{x^2-1}+\sqrt{x^2-2}}{\sqrt{x^2+1}+x}\right)$$

$$=\frac{1}{1}.\left(\frac{\sqrt{x^2-1}+\sqrt{x^2-2}}{\sqrt{x^2+1}+x}\right)$$

$$=\frac{\sqrt{x^2-1}+\sqrt{x^2-2}}{\sqrt{x^2+1}+x}$$

$$=\frac{|x|\sqrt{1-\frac{1}{x^2}}+|x|\sqrt{1-\frac{2}{x^2}}}{|x|\sqrt{1+\frac{1}{x^2}}+x}$$

$$=\frac{x\sqrt{1-\frac{1}{x^2}}+x\sqrt{1-\frac{2}{x^2}}}{x\sqrt{1+\frac{1}{x^2}}+x}$$

$$=\frac{x\left(\sqrt{1-\frac{1}{x^2}}+\sqrt{1-\frac{2}{x^2}}\right)}{x\left(\sqrt{1+\frac{1}{x^2}}+1\right)}$$

$$=\frac{\sqrt{1-\frac{1}{x^2}}+\sqrt{1-\frac{2}{x^2}}}{\sqrt{1+\frac{1}{x^2}}+1}$$

$$\lim_{x\to+\infty} f(x)=\frac{2}{2}=1$$

$f(x)=\sqrt{2x^2-1}-3x$ $a=+\infty$	$f(x)=\dfrac{3x^2-3}{x-1}\ ,a=1$
$f(x)=\sqrt{x^2-1}-x$ $a=+\infty$	$f(x)=\sqrt{3x^2+2x+2}-\sqrt{3x+1}$ $a=+\infty$
$f(x)=\sqrt{9x^2+2x-1}-2x+1$ $a=+\infty$	$f(x)=\dfrac{\sqrt{x^2+3}}{2x}$ $a=-\infty$
$f(x)=\dfrac{x}{\sqrt{x+1}}-\dfrac{x}{\sqrt{x+2}}$ $a=+\infty$	$f(x)=\dfrac{x\sqrt{x}-2\sqrt{2}}{\sqrt{x}-\sqrt{2}}$ $a=2\ ,+\infty$
$f(x)=\dfrac{\sqrt{x+1}-3}{16-2x}$ $a=8$	$f(x)=x^2\left(\sqrt{2+\dfrac{1}{x}}-\sqrt{2}\right)$ $a=+\infty$

$$f(x)=\frac{3x^2-3}{x-1}$$

$$\lim_{x\to 1} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{3x^2-3}{x-1}$$

$$=\frac{3(x^2-1)}{x-1}$$

$$=\frac{3(x-1)(x+1)}{x-1}$$

$$=3(x+1)$$

$$=3x+3$$

$$\lim_{x\to 1} f(x)=6$$

$$f(x)=\sqrt{2x^2-1}-3x$$

$$\lim_{x\to+\infty} f(x)$$

وهي حالة عدم تعيين من الشكل $\infty-\infty$

$$f(x)=\sqrt{2x^2-1}-3x$$

$$=|x|\sqrt{2-\frac{1}{x^2}}-3x$$

$$=x\sqrt{2-\frac{1}{x^2}}-3x$$

$$=x\left(\sqrt{2-\frac{1}{x^2}}-3\right)$$

$$\lim_{x\to+\infty} f(x)=+\infty(-1)=-\infty$$

$$f(x)=\sqrt{3x^2+2x+2}-\sqrt{3x+1}$$

$$\lim_{x\to+\infty} f(x)$$

وهي حالة دم تعيين من الشكل $\infty-\infty$

$$f(x)=\sqrt{3x^2+2x+2}-\sqrt{3x+1}$$

$$=|x|\sqrt{3+\frac{2}{x}+\frac{2}{x^2}}-|x|\sqrt{\frac{3}{x}+\frac{1}{x^2}}$$

$$=x\sqrt{3+\frac{2}{x}+\frac{2}{x^2}}-x\sqrt{\frac{3}{x}+\frac{1}{x^2}}$$

$$=x\left(\sqrt{3+\frac{2}{x}+\frac{2}{x^2}}-\sqrt{\frac{3}{x}+\frac{1}{x^2}}\right)$$

$$\lim_{x\to+\infty} f(x)=+\infty\left(\sqrt{3}\right)=+\infty$$

$$f(x)=\sqrt{x^2-1}-x$$

$$\lim_{x\to+\infty} f(x)$$

وهي حالة عدم تعيين من الشكل $\infty-\infty$

$$f(x)=\sqrt{x^2-1}-x$$

$$=\frac{\left(\sqrt{x^2-1}-x\right)\left(\sqrt{x^2-1}+x\right)}{\sqrt{x^2-1}+x}$$

$$=\frac{x^2-1-x^2}{\sqrt{x^2-1}+x}$$

$$=\frac{-1}{\sqrt{x^2-1}+x}$$

$$\lim_{x\to+\infty} f(x)=\frac{-1}{\infty}=0$$

$$f(x)=\frac{\sqrt{x^2+3}}{2x}$$

$$\lim_{x\to-\infty} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{\infty}{\infty}$

$$f(x)=\frac{\sqrt{x^2+3}}{2x}$$

$$=\frac{|x|\sqrt{1+\frac{3}{x^2}}}{2x}$$

$$=\frac{-x\sqrt{1+\frac{3}{x^2}}}{2x}$$

$$=\frac{-\sqrt{1+\frac{3}{x^2}}}{2}$$

$$\lim_{x\to-\infty} f(x)=-\frac{1}{2}$$

$$f(x)=\sqrt{9x^2+2x-1}-2x+1$$

$$\lim_{x\to+\infty} f(x)$$

وهي حالة عدم تعيين من الشكل $\infty-\infty$

$$f(x)=\sqrt{9x^2+2x-1}-2x+1$$

$$=|x|\sqrt{9+\frac{2}{x}-\frac{1}{x^2}}-2x+1$$

$$=x\sqrt{9+\frac{2}{x}-\frac{1}{x^2}}-2x+1$$

$$=x\left(\sqrt{9+\frac{2}{x}-\frac{1}{x^2}}-2+\frac{1}{x}\right)$$

$$\lim_{x\to+\infty} f(x)=+\infty(1)=+\infty$$

$$f(x)=\frac{x\sqrt{x}-2\sqrt{2}}{\sqrt{x}-\sqrt{2}}$$

إيجاد النهاية عند الـ 2 :

$$\lim_{x\to 2} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{x\sqrt{x}-2\sqrt{2}}{\sqrt{x}-\sqrt{2}}$$

$$=\frac{\left(\sqrt{x}\right)^3-\left(\sqrt{2}\right)^3}{\sqrt{x}-\sqrt{2}}$$

$$=\frac{\left(\sqrt{x}-\sqrt{2}\right)\left(x+\sqrt{2}\sqrt{x}+2\right)}{\sqrt{x}-\sqrt{2}}$$

$$=x+\sqrt{2x}+2$$

$$\lim_{x\to 2} f(x)=2+2+2=6$$

إيجاد النهاية عند الـ $+\infty$:

$$\lim_{x\to+\infty} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{\infty}{\infty}$

$$f(x)=\frac{x\sqrt{x}-2\sqrt{2}}{\sqrt{x}-\sqrt{2}}$$

$$=\frac{\sqrt{x}\left(x-\frac{2\sqrt{2}}{\sqrt{x}}\right)}{\sqrt{x}\left(1-\frac{\sqrt{2}}{\sqrt{x}}\right)}$$

$$=\frac{x-\frac{2\sqrt{2}}{\sqrt{x}}}{1-\frac{\sqrt{2}}{\sqrt{x}}}$$

$$\lim_{x\to+\infty} f(x)=+\infty$$

$$f(x)=\frac{x}{\sqrt{x+1}}-\frac{x}{\sqrt{x+2}}$$

$$\lim_{x\to+\infty} f(x)$$

وهي حالة عدم تعيين

$$f(x)=\frac{x}{\sqrt{x+1}}-\frac{x}{\sqrt{x+2}}$$

$$=\frac{x\sqrt{x+2}-x\sqrt{x+1}}{\sqrt{x+1}.\sqrt{x+2}}$$

$$=\frac{x\left(\sqrt{x+2}-\sqrt{x+1}\right)}{\sqrt{(x+1)(x+2)}}$$

$$=\frac{x}{\sqrt{x^2+3x+2}}.\left(\sqrt{x+2}-\sqrt{x+1}\right)$$

$$=\frac{x}{\sqrt{x^2+3x+2}}.\left(\frac{\left(\sqrt{x+2}-\sqrt{x+1}\right)\left(\sqrt{x+2}+\sqrt{x+1}\right)}{\sqrt{x+2}+\sqrt{x+1}}\right)$$

$$=\frac{x}{\sqrt{x^2+3x+2}}.\left(\frac{x+2-x-1}{\sqrt{x+2}+\sqrt{x+1}}\right)$$

$$=\frac{x}{\sqrt{x^2+3x+2}}.\left(\frac{1}{\sqrt{x+2}+\sqrt{x+1}}\right)$$

$$=\frac{x}{|x|\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}}.\left(\frac{1}{\sqrt{x+2}+\sqrt{x+1}}\right)$$

$$f(x)=\frac{x}{x\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}}.\left(\frac{1}{\sqrt{x+2}+\sqrt{x+1}}\right)$$

$$=\frac{1}{\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}\left(\sqrt{x+2}+\sqrt{x+1}\right)}$$

$$\lim_{x\to+\infty} f(x)=\frac{1}{\infty}=0$$

$$f(x) = x^2\left(\sqrt{2+\frac{1}{x}}-\sqrt{2}\right)$$

$$\lim_{x\to+\infty} f(x)$$

وهي حالة عدم تعيين من الشكل $0(\infty)$

$$f(x) = x^2\left(\sqrt{2+\frac{1}{x}}-\sqrt{2}\right)$$

$$= x^2 . \left(\frac{\left(\sqrt{2+\frac{1}{x}}-\sqrt{2}\right)\left(\sqrt{2+\frac{1}{x}}+\sqrt{2}\right)}{\sqrt{2+\frac{1}{x}}+\sqrt{2}}\right)$$

$$= x^2 . \left(\frac{2+\frac{1}{x}-2}{\sqrt{2+\frac{1}{x}}+\sqrt{2}}\right)$$

$$= \frac{x^2\left(\frac{1}{x}\right)}{\sqrt{2+\frac{1}{x}}+\sqrt{2}}$$

$$= \frac{x}{\sqrt{2+\frac{1}{x}}+\sqrt{2}}$$

$$\lim_{x\to+\infty} f(x) = +\infty$$

$$f(x) = \frac{\sqrt{x+1}-3}{16-2x}$$

$$\lim_{x\to 8} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x) = \frac{\left(\sqrt{x+1}-3\right)\left(\sqrt{x+1}+3\right)}{(16-2x)\left(\sqrt{x+1}+3\right)}$$

$$= \frac{x+1-9}{(16-2x)\left(\sqrt{x+1}+3\right)}$$

$$= \frac{x-8}{16-2x} . \left(\frac{1}{\sqrt{x+1}+3}\right)$$

$$= \frac{x-8}{-2(x-8)} . \left(\frac{1}{\sqrt{x+1}+3}\right)$$

$$= -\frac{1}{2} . \left(\frac{1}{\sqrt{x+1}+3}\right)$$

$$\lim_{x\to 8} f(x) = -\frac{1}{2}\left(\frac{1}{6}\right) = -\frac{1}{12}$$

$f(x) = \sin x\sqrt{1+\frac{1}{x^2}}\ , a = 0^+$	$f(x) = \frac{x^2 - x}{\sin x}\ , a = 0$
$f(x) = \frac{\sin(2x)}{3x}\ , a = 0$	$f(x) = \frac{\sin(\pi\sqrt{1-x})}{x}\ , a = 0$
$f(x) = \frac{\sin(\pi x)}{\sin(2x)}\ , a = 0$	$f(x) = \frac{\sin x}{\sqrt{x^2+x}}\ , a = 0^+$
$f(x) = \frac{2x^2 + \sin x}{x}\ , a = 0$	$f(x) = \frac{\sqrt{x^2+4}-2}{x.\sin 3x}\ , a = 0$

$$f(x) = \sin x\sqrt{1+\frac{1}{x^2}}$$

$$\lim_{x\to 0^+} f(x)$$

وهي حالة عدم تعيين من الشكل $0(\infty)$

$$f(x) = \sin x\sqrt{1+\frac{1}{x^2}}$$

$$= \sin x\sqrt{\frac{x^2+1}{x^2}}$$

$$= \sin x\frac{\sqrt{x^2+1}}{\sqrt{x^2}}$$

$$= \sin x\frac{\sqrt{x^2+1}}{|x|}$$

بما أن $x \to 0^+$ فإن $|x| = x$

$$= \frac{\sin x}{x}.\sqrt{x^2+1}$$

$$\lim_{x\to 0^+} f(x) = (1)(1) = 1$$

علماً أن:

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1$$

$$f(x) = \frac{x^2 - x}{\sin x}$$

$$\lim_{x\to 0} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x) = \frac{x^2 - x}{\sin x}$$

$$= \frac{x(x-1)}{\sin x}$$

$$= \frac{x}{\sin x}.(x-1)$$

$$\lim_{x\to 0} f(x) = 1(-1) = -1$$

علماً أن:

$$\lim_{x\to 0}\left(\frac{x}{\sin x}\right) = 1$$

$$f(x)=\frac{\sin 2x}{3x}$$

$$\lim_{x\to 0} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{\sin 2x}{3x}$$

$$=\frac{1}{3}\cdot\frac{\sin 2x}{x}$$

$$=\frac{2}{3}\cdot\frac{\sin 2x}{2x}$$

$$\lim_{x\to 0} f(x)=\frac{2}{3}(1)=\frac{2}{3}$$

علماً أن:

$$\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)=1$$

$$f(x)=\frac{\sin(\pi x)}{\sin(2x)}$$

$$\lim_{x\to 0} f(x)$$

وهي حالة عد تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{\sin(\pi x)}{\sin(2x)}$$

$$=\frac{\sin(\pi x)}{1}\cdot\frac{1}{\sin(2x)}$$

$$=\frac{\sin \pi x}{x}\cdot\frac{x}{\sin(2x)}$$

$$=\frac{\pi}{2}\cdot\frac{\sin \pi x}{\pi x}\cdot\frac{2x}{\sin 2x}$$

$$=\frac{\pi}{2}\cdot\frac{\sin \pi x}{\pi x}\cdot\frac{2x}{\sin 2x}$$

$$\lim_{x\to 0} f(x)=\frac{\pi}{2}(1)(1)=\frac{\pi}{2}$$

علماً أن:

$$\lim_{x\to 0}\left(\frac{\sin \pi x}{\pi x}\right)=\lim_{x\to 0}\left(\frac{2x}{\sin 2x}\right)=1$$

$$f(x)=\frac{\sin x}{\sqrt{x^2+x}}$$

$$\lim_{x\to 0^+} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{\sin x}{\sqrt{x^2+x}}$$

$$=\frac{\sin x}{|x|\sqrt{1+\frac{1}{x}}}$$

بما أن $x\to 0^+$ فإن $|x|=x$

$$=\frac{\sin x}{x\sqrt{1+\frac{1}{x}}}=\frac{\sin x}{x}\cdot\frac{1}{\sqrt{1+\frac{1}{x}}}$$

$$\lim_{x\to 0^+} f(x)=(1)(0)=0$$

علماً أن:

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1$$

$$f(x)=\frac{2x^2+\sin x}{x}$$

$$\lim_{x\to 0} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{2x^2+\sin x}{x}$$

$$=\frac{2x^2}{x}+\frac{\sin x}{x}=2x+\frac{\sin x}{x}$$

$$\lim_{x\to 0} f(x)=0+1=1$$

علماً أن:

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1$$

$$f(x)=\frac{\sin\left(\pi\sqrt{1-x}\right)}{x}$$

$$\lim_{x\to 0} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{\sin\left(\pi\sqrt{1-x}\right)}{x}$$

نعلم أن: $\sin\theta=\sin(\pi-\theta)$

ومنه:

$$\sin\left(\pi\sqrt{1-x}\right)=\sin\left(\pi-\pi\sqrt{1-x}\right)$$

$$f(x)=\frac{\sin\left(\pi-\pi\sqrt{1-x}\right)}{x}$$

$$=\frac{\sin\left(\pi\left(1-\sqrt{1-x}\right)\right)}{x}$$

$$=\frac{\sin\left(\pi\left(1-\sqrt{1-x}\right)\right)}{1}\cdot\frac{1}{x}$$

$$=\frac{\sin\left(\pi\left(1-\sqrt{1-x}\right)\right)}{\pi\left(1-\sqrt{1-x}\right)}\cdot\frac{\pi\left(1-\sqrt{1-x}\right)}{x}$$

$$=\frac{\sin\left(\pi\left(1-\sqrt{1-x}\right)\right)}{\pi\left(1-\sqrt{1-x}\right)}\cdot\frac{\pi}{x}\frac{\left(1-\sqrt{1-x}\right)\left(1+\sqrt{1-x}\right)}{\left(1+\sqrt{1-x}\right)}$$

$$=\frac{\sin\left(\pi\left(1-\sqrt{1-x}\right)\right)}{\pi\left(1-\sqrt{1-x}\right)}\cdot\frac{\pi}{x}\frac{(1-1+x)}{\left(1+\sqrt{1-x}\right)}$$

$$=\frac{\sin\left(\pi\left(1-\sqrt{1-x}\right)\right)}{\pi\left(1-\sqrt{1-x}\right)}\cdot\frac{\pi}{x}\frac{x}{\left(1+\sqrt{1-x}\right)}$$

$$=\frac{\sin\left(\pi\left(1-\sqrt{1-x}\right)\right)}{\pi\left(1-\sqrt{1-x}\right)}\cdot\frac{\pi}{\left(1+\sqrt{1-x}\right)}$$

$$\lim_{x\to 0} f(x)=(1).\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$$

علماً أن:

$$\lim_{x\to 0}\left(\frac{\sin\left(\pi\left(1-\sqrt{1-x}\right)\right)}{\pi\left(1-\sqrt{1-x}\right)}\right)=1$$

$$f(x)=\frac{\sqrt{x^2+4}-2}{x\sin 3x}$$

$$\lim_{x\to 0} f(x)$$

وهي حالة عدم تعيين من الشكل $\frac{0}{0}$

$$f(x)=\frac{\sqrt{x^2+4}-2}{x\sin 3x}$$

$$=\frac{\left(\sqrt{x^2+4}-2\right)\left(\sqrt{x^2+4}+2\right)}{x\sin 3x\left(\sqrt{x^2+4}+2\right)}$$

$$=\frac{x^2+4-4}{x\sin 3x\left(\sqrt{x^2+4}+2\right)}$$

$$=\frac{x^2}{x\sin 3x\left(\sqrt{x^2+4}+2\right)}$$

$$=\frac{x}{\sin 3x}\cdot\frac{1}{\sqrt{x^2+4}+2}$$

$$=\frac{3x}{\sin 3x}\cdot\frac{1}{3\left(\sqrt{x^2+4}+2\right)}$$

$$\lim_{x\to 0} f(x)=1.\left(\frac{1}{12}\right)=\frac{1}{12}$$

علماً أن:

$$\lim_{x\to 0}\left(\frac{3x}{\sin 3x}\right)=1$$

انتهى السؤال الأول ^_^

التمرين الثاني:

ليكن لدينا التابع f المعين بالعلاقة:

$$f(x) = \frac{x+3}{x-3}$$

١. أوجد نهاية التابع f عند الـ 5

٢. عين مجالاً I مركزه 5 يحقق الشرط: إذا كان x من المجال I كان $f(x)$ من المجال $J =]3.95,4.05[$

الطلب الأول:

$$\lim_{x\to 5} f(x) = \frac{8}{2} = 4$$

الطلب الثاني:

بما أن التابع f ينتمي إلى المجال J فإن:

$$3.95 < f(x) < 4.05$$

$$3.95 < \frac{x+3}{x-3} < 4.05$$

نصلح التابع f باستخدام القسم الاقليدية لكي نكتبه بدلالة x واحدة ومنه:

$$3.95 < 1 + \frac{6}{x-3} < 4.05$$

$$2.95 < \frac{6}{x-3} < 3.05$$

$$\frac{1}{2.95} > \frac{x-3}{6} > \frac{1}{3.05}$$

$$\frac{6}{2.95} > x-3 > \frac{6}{3.05}$$

$$\frac{6}{2.95} + 3 > x > \frac{6}{3.05} + 3$$

$$5.03 > x > 4.97$$

إذاً:

$$I =]4.97,5.03[$$

التمرين الثالث:

ليكن لدينا التابع f المعرف على $]-5,+\infty[$ وفق:

$$f(x) = \frac{2x+1}{x+5}$$

1. احسب $\lim_{x\to+\infty} f(x)$ واستنتج $\lim_{x\to+\infty} f(f(x))$

2. جد عدداً حقيقياً يحقق الشرط: إذا كان $x > A$ كان $f(x)$ من المجال $]1.99,2.01[$

الطلب الأول:

$$\lim_{x\to+\infty} f(x) = 2$$

استنتاج $\lim_{x\to+\infty} f(f(x))$

$$\lim_{x\to+\infty} f(x) = 2$$

$$\lim_{x\to 2} f(x) = \frac{5}{7}$$

$$\lim_{x\to+\infty} f(f(x)) = \frac{5}{7}$$

الطلب الثاني:

نحدد مركز ونصف قطر المجال وفق:

$$C = \frac{a+b}{2} = \frac{1.99+2.01}{2} = \frac{4}{2} = 2$$

$$r = \frac{b-a}{2} = \frac{2.01-1.99}{2}$$

$$= \frac{0.02}{2} = \frac{2}{200} = \frac{1}{100} = 0.01$$

بما أن $f(x)$ من المجال $]1.99,2.01[$ فإن:

$$|f(x) - C| < r$$

$$\left|\frac{2x+1}{x+5} - 2\right| < \frac{1}{100}$$

$$\left|\frac{2x+1-2x-10}{x+5}\right| < \frac{1}{100}$$

$$\left|\frac{-9}{x+5}\right| < \frac{1}{100}$$

$$\frac{|-9|}{|x+5|} < \frac{1}{100}$$

$$\frac{9}{x+5} < \frac{1}{100}$$

$$\frac{x+5}{9} > 100$$

$$x+5 > 900$$

$$x > 895 \to A \geq 895$$