

$$u_{n+1} = \frac{6u_n - 4}{u_n + 1}; u_0 = 2$$

$$v_n = \frac{u_n - 4}{u_n - 1}$$

نوجد v_{n+1} :

$$v_{n+1} = \frac{u_{n+1} - 4}{u_{n+1} - 1} = \frac{\frac{6u_n - 4}{u_n + 1} - 4}{\frac{6u_n - 4}{u_n + 1} - 1} = \frac{\frac{6u_n - 4 - 4u_n - 4}{u_n + 1}}{\frac{6u_n - 4 - u_n - 1}{u_n + 1}} = \frac{2u_n - 8}{5u_n - 5} = \frac{2}{5} \left(\frac{u_n - 4}{u_n - 1} \right)$$

نشكل النسبة:

$$\frac{v_{n+1}}{v_n} = \frac{\frac{2}{5} \left(\frac{u_n - 4}{u_n - 1} \right)}{\left(\frac{u_n - 4}{u_n - 1} \right)} = \frac{2}{5} = q$$

هندسية أساسها $q = \frac{2}{5}$.

الحد الأول:

$$v_0 = \frac{u_0 - 4}{u_0 - 1} = \frac{2 - 4}{2 - 1} = -2$$

الحد العام:

$$v_n = v_0 \cdot q^n = -2 \left(\frac{2}{5} \right)^n = -\frac{2^{n+1}}{5^n}$$

الحد العام للمتتالية u_n , أولاً سنوجد u_n بدلالة v_n :

$$v_n = \frac{u_n - 4}{u_n - 1}$$

$$v_n(u_n - 1) = u_n - 4$$

$$v_n u_n - v_n = u_n - 4$$

$$v_n u_n - u_n = v_n - 4$$

$$u_n(v_n - 1) = v_n - 4$$

$$u_n = \frac{v_n - 4}{v_n - 1}$$

$$= \frac{-\frac{2^{n+1}}{5^n} - 4}{-\frac{2^{n+1}}{5^n} - 1} = \frac{\frac{-2^{n+1} - 4 \cdot 5^n}{5^n}}{\frac{-2^{n+1} - 5^n}{5^n}} = \frac{-(2^{n+1} + 4 \cdot 5^n)}{-(2^{n+1} + 5^n)} = \frac{2^{n+1} + 4 \cdot 5^n}{2^{n+1} + 5^n}$$

$$u_n = \frac{-2\left(\frac{2}{5}\right)^n - 4}{-2\left(\frac{2}{5}\right)^n - 1} = \frac{-\left(2\left(\frac{2}{5}\right)^n + 4\right)}{-\left(2\left(\frac{2}{5}\right)^n + 1\right)} = \frac{2\left(\frac{2}{5}\right)^n + 4}{2\left(\frac{2}{5}\right)^n + 1}$$

